

# Faculty of Science Course Syllabus

## Department of Mathematics and Statistics

### Math 4070/5070

### Algebraic Number Theory

### Winter 2021

#### Instructor

Instructor	E-mail Address	Office Hours Information
Rob Noble	<a href="mailto:rnoble@mathstat.dal.ca">rnoble@mathstat.dal.ca</a>	By E-mail

#### Lectures

Asynchronous: Beamer (PowerPoint like) presentations will be posted on BrightSpace three times per week, once on each of Monday, Wednesday and Friday, and will be designed to break down the material from the course notes into “lecture-sized” pieces.

#### Course Description

An introduction to algebraic number theory, with special emphasis on quadratic and cyclotomic fields. A more general study of rings of integers of algebraic number fields focuses on divisibility properties. Other topics include Dedekind domains, ideals and their factorization into prime ideals, and class groups and class numbers.

#### Course Prerequisites

MATH 3032.03 (MATH 3070.03 recommended)

#### Learning Objectives

The main goal of this course is to provide an algebraic proof of Gauss’ Law of Quadratic Reciprocity, using Galois Theory. This will require the use of most of the material presented in the course, detailed below in the “Course Content” section.

#### Course Materials

- Textbook: Algebraic theory of numbers, by Pierre Samuel ([Sam08]).
- Course notes (as well as all other important course materials) will be made available on BrightSpace.

## Course Assessment

The Final Grade will be computed as follows:

Component	Weight (% of final grade)	Date
“Take-Home” Midterm Exam	35%	February 24 – February 26
“Take-Home” Final Exam	35%	(Scheduled by Registrar)
Assignments	30%	Weekly

**Note:** Each of the assignments, the midterm and the final exam are to be written, with full solutions and submitted via e-mail, preferably as a single PDF file.

Conversion of numerical grades to Final Letter Grades follows the Dalhousie Common Grade Scale

A+: (90–100)	A: (85–89)	A-: (80–84)
B+: (77–79)	B: (73–76)	B-: (70–72)
C+: (65–69)	C: (60–64)	C-: (55–59)
D: (50–54)	F: (<50)	

## Course Policies

- Missed assignments, midterms or final exams can be made up for documented illness / inability to participate at the given time.
- Collaboration during the midterm or during the final exam **is not allowed**. These components of the course **must** be completed independently.
- Students may collaborate on assignments, but must write up their own solutions, in their own words.

## Course Content

The contents of the provided course notes will be covered. They consist of the majority of the sections of the textbook:

- Notations, definitions, and prerequisites
- Chapter I: Principal ideal rings
  - I.1 Divisibility in principal ideal rings
  - I.3 Some lemmas concerning ideals; Euler’s  $\varphi$ -function
  - I.4 Some preliminaries concerning modules
  - I.5 Modules over principal ideal rings
  - I.6 Roots of unity in a field
  - I.7 Finite fields

- Chapter II: Elements integral over a ring; elements algebraic over a field

- II.1 Elements integral over a ring
- II.2 Integrally closed rings
- II.3 Elements algebraic over a field. Algebraic extensions
- II.4 Conjugate elements, conjugate fields
- II.5 Integers in quadratic fields
- II.6 Norms and traces
- II.7 The discriminant
- II.8 The terminology of number fields
- II.9 Cyclotomic fields

- Chapter III: Noetherian rings and Dedekind rings

- III.1 Noetherian rings and modules
- III.2 An application concerning integral elements
- III.3 Some preliminaries concerning ideals
- III.4 Dedekind rings
- III.5 The norm of an ideal

- Chapter IV: Ideal classes and the unit theorem

- IV.1 Preliminaries concerning discrete subgroups of  $\mathbb{R}^n$
- IV.2 The canonical embedding of a number field
- IV.3 Finiteness of the ideal class group
- IV.4 The unit theorem
- IV.5 Units in imaginary quadratic fields
- IV.6 Units in real quadratic fields

- Chapter V: The splitting of prime ideals in an extension field

- V.1 Preliminaries concerning rings of fractions
- V.2 The splitting of a prime ideal in an extension
- V.3 The discriminant and ramification
- V.4 The splitting of a prime number in a quadratic field
- V.6 The two-squares theorem

- Chapter VI: Galois extensions of number fields

- VI.1 Galois theory
- VI.2 The decomposition and inertia groups
- VI.3 The number field case. The Frobenius automorphism
- VI.4 An application to cyclotomic fields
- VI.5 Another proof of the quadratic reciprocity law

## University Policies and Statements

This course is governed by the academic rules and regulations set forth in the University Calendar and by Senate.

### Missed or Late Academic Requirements due to Student Absence

As per Senate decision instructors may not require medical notes of students who must miss an academic requirement, including the final exam, for courses offered during fall or winter 2020-21 (until April 30, 2021). Information on regular policy, including the use of the Student Declaration of Absence can be found here:

[https://www.dal.ca/dept/university\\_secretariat/policies/academic/missed-or-late-academic-requirements-due-to-student-absence.html](https://www.dal.ca/dept/university_secretariat/policies/academic/missed-or-late-academic-requirements-due-to-student-absence.html)

### Academic Integrity

At Dalhousie University, we are guided in all of our work by the values of academic integrity: honesty, trust, fairness, responsibility and respect (The Center for Academic Integrity, Duke University, 1999). As a student, you are required to demonstrate these values in all of the work you do. The University provides policies and procedures that every member of the university community is required to follow to ensure academic integrity.

#### Information:

[https://www.dal.ca/dept/university\\_secretariat/academic-integrity.html](https://www.dal.ca/dept/university_secretariat/academic-integrity.html)

### Accessibility

The Advising and Access Services Centre is Dalhousie's centre of expertise for student accessibility and accommodation. The advising team works with students who request accommodation as a result of a disability, religious obligation, or any barrier related to any other characteristic protected under Human Rights legislation (Canada and Nova Scotia).

#### Information:

[https://www.dal.ca/campus\\_life/academic-support/accessibility.html](https://www.dal.ca/campus_life/academic-support/accessibility.html)

### Student Code Of Conduct

Everyone at Dalhousie is expected to treat others with dignity and respect. The Code of Student Conduct allows Dalhousie to take disciplinary action if students don't follow this community expectation. When appropriate, violations of the code can be resolved in a reasonable and informal manner-perhaps through a restorative justice process. If an informal resolution can't be reached, or would be inappropriate, procedures exist for formal dispute resolution.

#### Code:

[https://www.dal.ca/dept/university\\_secretariat/policies/student-life/code-of-student-conduct.html](https://www.dal.ca/dept/university_secretariat/policies/student-life/code-of-student-conduct.html)

## Diversity and Inclusion - Culture of Respect

Every person at Dalhousie has a right to be respected and safe. We believe inclusiveness is fundamental to education. We stand for equality. Dalhousie is strengthened in our diversity. We are a respectful and inclusive community. We are committed to being a place where everyone feels welcome and supported, which is why our Strategic Direction prioritizes fostering a culture of diversity and inclusiveness.

### Statement:

<http://www.dal.ca/cultureofrespect.html>

## Recognition of Mi'kmaq Territory

Dalhousie University would like to acknowledge that the University is on Traditional Mi'kmaq Territory. The Elders in Residence program provides students with access to First Nations elders for guidance, counsel and support. Visit or e-mail the Indigenous Student Centre (1321 Edward St) ([elders@dal.ca](mailto:elders@dal.ca)).

### Information:

[https://www.dal.ca/campus\\_life/communities/indigenous.html](https://www.dal.ca/campus_life/communities/indigenous.html)

## Important Dates in the Academic Year (including add/drop dates)

[https://www.dal.ca/academics/important\\_dates.html](https://www.dal.ca/academics/important_dates.html)

## University Grading Practices

[https://www.dal.ca/dept/university\\_secretariat/policies/academic/grading-practices-policy.html](https://www.dal.ca/dept/university_secretariat/policies/academic/grading-practices-policy.html)

## Student Resources and Support

### Advising

**General Advising:** [https://www.dal.ca/campus\\_life/academic-support/advising.html](https://www.dal.ca/campus_life/academic-support/advising.html)

**Science Program Advisors:** <https://www.dal.ca/faculty/science/current-students/academic-advising.html>

**Indigenous Student Centre:** [https://www.dal.ca/campus\\_life/communities/indigenous.html](https://www.dal.ca/campus_life/communities/indigenous.html)

**Black Advising Centre:** [https://www.dal.ca/campus\\_life/communities/black-student-advising.html](https://www.dal.ca/campus_life/communities/black-student-advising.html)

**International Centre:** [https://www.dal.ca/campus\\_life/international-centre/current-students.html](https://www.dal.ca/campus_life/international-centre/current-students.html)

### Academic supports

Library: <https://libraries.dal.ca/>

Writing Centre: [https://www.dal.ca/campus\\_life/academic-support/writing-and-study-skills.html](https://www.dal.ca/campus_life/academic-support/writing-and-study-skills.html)

Studying for Success: [https://www.dal.ca/campus\\_life/academic-support/study-skills-and-tutoring.html](https://www.dal.ca/campus_life/academic-support/study-skills-and-tutoring.html)

Copyright Office: <https://libraries.dal.ca/services/copyright-office.html>

Fair Dealing Guidelines <https://libraries.dal.ca/services/copyright-office/fair-dealing.html>

## Other supports and services

Student Health & Wellness Centre: [https://www.dal.ca/campus\\_life/health-and-wellness/services-support/student-health-and-wellness.html](https://www.dal.ca/campus_life/health-and-wellness/services-support/student-health-and-wellness.html)

Student Advocacy: <https://dsu.ca/dsas>

Ombudsperson: [https://www.dal.ca/campus\\_life/safety-respect/student-rights-and-responsibilities/where-to-get-help/ombudsperson.html](https://www.dal.ca/campus_life/safety-respect/student-rights-and-responsibilities/where-to-get-help/ombudsperson.html)

## Safety

Biosafety: <https://www.dal.ca/dept/safety/programs-services/biosafety.html>

Chemical Safety: <https://www.dal.ca/dept/safety/programs-services/chemical-safety.html>

Radiation Safety: <https://www.dal.ca/dept/safety/programs-services/radiation-safety.html>

## Scent-Free Program

<https://www.dal.ca/dept/safety/programs-services/occupational-safety/scent-free.html>

## References

[Sam08] Pierre Samuel, *Algebraic theory of numbers*, Dover Publications, 2008.